An Investigation into The Application of Function Fitting in Trend Analysis

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Abstract. Trend information of discrete observations is an important input in science in data-driven environments. The following paper critically analyzes the usage value and risks of trend analysis with use of function fitting. To begin with, it classifies the five most popular fitting techniques, including linear, non-linear (polynomial), exponential, periodic, and power functions and it builds a method selection system ranging between easy and complicated. It then compares the effectiveness in various fitting schemes using stock prices, temperature changes, and drug pharmacokinetics as case studies to find the practical use of various fitting schemes in trend extraction, volatility smoothing and predictive accuracy enhancement. The results show that accuracy fitting depends on the synergies of the model complexity, data quality and domain knowledge and that overfitting and homoscedasticity and structure misidentification are important contributors to poor extrapolation reliability. The conclusion verifies that functional fitting is an important instrument that cannot be neglected in the trend analysis but its successful usage depends on correct perception of the peculiarities of the methods, strict control of the absence of data, and thorough combination with professional processes. The next efforts will involve machine learning and uncertainty quantification methods together in order to create a more robust and interpretable intelligent fitting algorithm, which can meet the needs of decision-making in high-dimensional complex data.

Keywords: Function Fitting; Trend Analysis; Application Scenarios.

1. Introduction

Nowadays, the size of the data and its dimensions have gone exponentially as big data becomes the order of the day. Deterministic trends are desired by decision-makers as they are required to obtain the trends, which are unrelated to noise, making the uncertainty cost less. Trend analysis has become a fundamental issue in interdisciplinary research due to this need. As the most important method of modelling discrete observations, to the extent that the applied mathematics of parameter estimation is regarded as offering a statistical framework by which the estimation of the parameters of the models can be performed and the complexity versus solvability of the model must be balanced by applied mathematics, functional fitting gives a statistical means of the estimation of the model parameters. It should also be able to use domain knowledge in order to satisfy physical interpretation needs. The outer developments of the methodology of the technique of progress of functions are an immediate expression of the gaining knowledge of uncertainty and complexity. On a practical level, high-precision trend results are needed in: the volatility of asset prices in financial forecasting; the development of extreme temperatures in climate change; the downslope point of an epidemic in epidemiology; and the evaluation of the remaining life of a piece of equipment in its maintenance. Any change of the role in the stage of fitting the functions increases the chain of the decision that causes incorrect allocation of assets, a collapse of the plan of preventing disasters, incorrect allocation of the resources of the public health, or unfortunate incidents of equipment shutdown. In this way, the improvement of the fitting reliability has become one of the essential conditions under the guarantee of the efficiency of the critical decisions. This paper is a systematic grouping of five frequently used fitting functions, and where these functions are applicable, in addition to analyzing common situations of application.

2. Common Function Types

Function fitting is a mathematical method that approximates data by selecting a functional form. Common forms include linear, polynomial, or exponential functions. It typically employs optimization algorithms like least squares to minimize the difference between the fitted curve and actual data points.

Trend analysis is a data analysis technique. It aims to identify long-term directional changes within time series or ordered data. Its defining feature is the ability to recognize trends such as upward or downward movements. It can also detect cyclical variations. Another characteristic is its capacity to reduce interference from short-term fluctuations.

Function fitting is well-suited for trend analysis. By fitting a function, a mathematical model linking data to time or sequence can be established. This model describes the overall direction of data change, thereby extracting long-term trends. The fitted function curve smooths out short-term fluctuations, making the core trend clearer. Therefore, function fitting is an effective tool for trend analysis.

Table 1 systematically outlines the key characteristics of five core function models in trend analysis.

Linear fitting can be directly applied when data exhibits a stable upward or downward trend.

If the curve shows significant curvature rather than a straight line, switch to polynomial fitting to capture nonlinear trends. Exponential or logarithmic fitting should be adopted when sustained acceleration or deceleration is observed. For sequences exhibiting recurring seasonal or cyclical fluctuations, periodic function fitting is appropriate. Finally, power function fitting is selected when the scale of a phenomenon varies proportionally with its numerical value, completing the full fitting process from simple to complex.

Table 1. Comparison of Common Function Fitting Methods in Trend Analysis

Function Type	Applicable Scenarios	Key Advantages and Disadvantages
Linear Fitting	Stable increasing or decreasing trends	Advantages: Simple model, clear parameter meaning, robust results. Disadvantages: Strict assumptions, poor flexibility.
Polynomial Fitting	Non-linear trends (e.g., changing growth rates)	Advantages: Flexible form, strong fitting capability. Disadvantages: Prone to overfitting with high orders.
Exponential / Logarithmic Fitting	Growth or decay trends (e.g., population, virus spread)	Advantages: Clear physical meaning of parameters. Disadvantages: Highly sensitive to initial values and growth rate parameters.
Periodic Function Fitting	Seasonal or cyclical fluctuations (e.g., temperature, sales)	Advantages: Highly effective for predicting periodic fluctuations. Disadvantages: Limited ability to handle non-stationary periods or complex multi-period overlaps.
Power Function Fitting	Scale-related phenomena (e.g., city size vs. GDP)	Advantages: Can reveal scaling laws and scale effects. Disadvantages: Requires high data completeness and a wide range of magnitudes.

3. Application Scenarios

3.1. Stock Price Trend Analysis

Linear regression serves as a fundamental tool for forecasting long-term stock trends. Trend lines fitted using the least squares method provide an intuitive representation of overall price movements. When stock prices exhibit nonlinear fluctuations, quadratic or cubic polynomials better capture their patterns, particularly during phase-specific market analysis.

Multiple studies have confirmed that decomposing stock price sequences into cyclical components via Fourier transforms, followed by constructing predictive models for residuals, significantly enhances short-term trend forecasting accuracy. For instance, Livieri et al. reconstructed the “volatility of volatility” for the S&P 500 using Fourier estimates. They found that after removing dominant cycles, the residual sequence's noise energy decreased by approximately 20%, leading to a 15% reduction in subsequent random forest prediction errors [1]. Traditional trend lines are essentially intuitive representations of piecewise linear fits. Modern software has digitized these hand-drawn lines—for instance, the Bloomberg Terminal's automated trendline feature employs multiple linear regression algorithms to provide institutional investors with quantitative references for support/resistance levels [2].

3.2. Temperature Variation Fitting

Three among the common and effective functional fitting methods that are commonly used and effective in temperature variation fit analysis are least squares method, polynomial fitting and sine function fitting. The least squares technique helps to minimize the total amount of squared errors to determine the functional relationship that best organizes the data trend to be used in either linear and nonlinear models. As an example, the least square fitting was used by the researchers to carry out six-curve fitting in temperature data acquisition in the capital region in the year 2009- 2010 and was successful in identifying seasonal temperatures. This technique was confirmed to be effective in the temperature fitting through comparative analysis with data of ground temperature [3]. Polynomial fitting is highly effective in terms of describing nonlinear relationship of temperature over time and is especially effective in situations where there are strong diurnal or seasonal changes. Also, the application of sine-function fitting fits in particular well when it comes to simulating daily or annual climate changes. As an example, Jiang Huifei et al. presented segmented sine method to represent daily curves of changes in temperatures. This method addressed the problem of lack of continuity between neighboring days of the traditional sine models very well. The comparison with real data provided by automatic weather stations proved that this technique could help to accurately restore the diurnal variability features of temperatures [4]. In short, least squares fitting, polynomial fitting and sine function fitting are some of the available approaches that have had general strengths and have been effectively used by various researchers in modeling and analysing temperature variation data. Zhong et al. used a cascaded CNN to learn the probability of surface temperatures in East Asia and reached a 23 percent reduction in the errors of 24-72hour price of extreme heat events of temperature compared to CMA-GEPS model [5].

3.3. Time-Dependent Decay in Drug Metabolism

Functional fitting serves as the core mathematical tool for quantifying drug concentration dynamics in time-dependent decay analysis. The exponential decay model represents the most classical approach. Johnson et al. employed a double-exponential decay model to fit intravenous ceftriaxone concentration data, successfully distinguishing distinct decay rates between the distribution and elimination phases, accurately calculating a half-life of 8.2 hours [6]. The WHO Antimalarial Drug Monitoring Program employed a nonlinear mixed-effects model (NONMEM) to fit 4217 plasma concentration data points of artemisinin (AS) and its active metabolite dihydroartemisinin (DHA) from 153 patients in Myanmar, Cambodia, and Thailand. This successfully quantified the individualized impact of hepatic and renal function on artemether clearance, providing quantitative evidence for dose adjustments in patients of different ages and with impaired hepatic or renal function [7].

4. Risks and Challenges

Function fitting has become a common trend analysis tool to demonstrate what patterns are present in variables through time, however, it has various risks and problems when using it. First, overfitting severely impairs an extrapolation ability of a model. In the case of the use of high-order polynomials or complex nonlinear functions, the model can attain a large determination coefficient (R2), but quickly inflates the error in the prediction of new data. The studies have shown that the higher the order of the poly, the better the curves of historical temperature are modeled, but the magnitude of the standard deviation of predication is borne up to more than 1.8 times the historical one with the help of a prediction of the following decade [8]. Second, the heteroscedasticity of data, and the non-random missingness give bias in the estimation of the parameters. With large blood samples mainly sampled during the absorption phase and little data on the elimination phase, nonlinear least squares do not estimate the elimination rate constant λ systematically with an error of about 15 percent [9]. Third, systematic errors are caused by the false model structure assumptions. An incorrectly applied single-exponential model being used when the processes are based on a two-exponential decay results in joint estimates of the distribution and elimination phase rate constants, which cause errors in the calculation of half-lives of up to 2030 per cent [10]. These dangers demonstrate the criticality of the overall consideration involved in the application of functional fitting in trend analysis.

5. Conclusion

The paper is a systematic review of the major techniques, common uses, and some of the challenges that may arise when conducting functional fitting in trend analysis. It is proven that the use of function fitting is an effective instrument that helps to discover the long-term dynamics and flatten the short-term changes through the transformations of discrete observations. The five different types of core models include linear models, polynomial models, exponential/logarithmic models, periodic models, power models with each having its own peculiarities, applicable to regular, nonlinear, growth/decay, cycle models and scale-related issues respectively, a complete approach to less complex (linear models) and more complex (nonlinear models) issues. Appropriate fitting techniques have shown in certain areas such as the analysis of stock prices, models of simulating climate change, and the pharmacokinetics of drugs, to improve trend identification accuracy and prediction accuracy, as the basis of quantitative financial decision reaction, climate analysis, and, generally, in clinical drug applications.

This paper synthesizes and elucidates the availability and restrictions of the various functional fitting approaches, with the point that the choice of a model should be firmly linked to the nature of data, and also to the a priori information in the domain. Throughout the research, it is revealed that even with the potent functions of the function fitting, its use continues being vulnerable to various dangers including overfitting, defects in data quality, maladaptation of the model and more. Such risks can be increased in real predictions, which results in decision biases. Consequently, mathematical goodness-of-fit should not be the only aspect pursued in the trend analysis: interpretation ability, strength, and extrapolation are also important to consider.

In the future, the development of the functional fitting approaches will become more intertwined with the latest technologies, including machine learning and uncertainty quantification. Moreover, creating smart fitting algorithms that can adapt to different situations and balance the complexity with generalization performance are going to play a central role in improving the reliability of trend analysis and helping to make the scientific decisions.

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